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A Geometric Look on the Microstates of Supertubes

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We give a geometric interpretation of the entropy of the supertubes with fixed conserved charges and angular momenta in two different approaches using the DBI action and the supermembrane theory. By counting the geometrically allowed microstates, it is shown that both the methods give consistent result on the entropy. In doing so, we make the connection to the gravity microstates clear.

1 Introduction

Supertubes are tubular shaped bound states of D0-branes, fundamental strings (F1) and D2-branes [1]-[30]. While having translational invariance in the axial direction along which the F1 strings are stretched, the cross sectional shape of the supertubes may be arbitrary in the eight transverse dimensions. As shown in Ref. [6], the cross sectional shape could be either open and stretched to infinity or closed but here we would like to focus on the closed cases.

Let us begin our discussion with the cases where the cross sectional curve lies in x^1 and x^2 plane. The supertube then carries an angular momentum density $J = J_{12}$ proportional to the cross sectional area. For the fixed conserved charges, the moduli space of supertubes is consisting of the geometric fluctuations of the cross sectional shape [27]. Since the angular momentum is fixed, the fluctuation of the curve has to be area preserving. The length L of the cross sectional curve is further limited by $\sqrt{Q_0 Q_1}$ where Q_0 and Q_1 denote lineal D0 density in the axis direction and F1 charges divided by 2π respectively. Thus one has the restriction of the length by $\sqrt{J/T_2} \leq L \leq \sqrt{Q_0 Q_1/T_2}$, where T_2 is the D2-brane tension. This space of arbitrary fluctuation of the curve forms an infinite dimensional moduli space. For given curve, the magnetic field representing the density of D0 may be arbitrary with total number of D0-branes fixed. Moreover the shape may fluctuate into the six more transverse directions. Hence eight arbitrary bosonic functional fluctuations are involved as the moduli deformation. Since the supertubes involve a nonvanishing electric field and linear momentum densities fixed by the shape of the curve, the above moduli space is not a configuration space but a phase space.

The supertubes allow corresponding supergravity solutions [4, 9] of an arbitrary cross sectional shape and arbitrary density of D0-brane as a function of the world-volume coordinate ϕ of the curve direction. Therefore the solution involves the same number of arbitrary functions of bosonic degrees. The geometry is nonsingular everywhere as argued in Ref. [31] in the U-dual picture. The solution does not have a horizon either. The recently emerging picture is that such a regular, no-horizon solution corresponds to distinguishable gravity microstates represented by supergravity fields [32]. When all the conserved charges and certain asymptotic conditions on the geometries are fixed, the logarithm of the number of above microstates is the entropy of the gravity system with certain macroscopic parameters fixed. In case of supergravity supertubes, we are interested in all the supersymmetric solutions with fixed energy, D0 and F1 charges and the angular momenta. The system may have many components of angular momenta of $SO(8)$ since the system involves eight transverse dimensions. We fix here the four independent Cartan elements of $SO(8)$.

This solution space with all the macroscopic conserved quantities fixed, forms a moduli space of the supergravity supertubes. As we see in the case of DBI description of supertubes, this space must be a phase space instead of a configuration space. Since the phase moduli space involves arbitrary functions, it is an infinite dimensional space. Hence its volume divided by $(2\pi\hbar)^{\dim}$ is either zero or infinity. Consequently it requires at least a regularization procedure.

By quantization, the above problem may be avoided but this will not be that simple since a direct quantization of gravity is not well defined as we know very well.

But the two sides of the system has a striking similarity in its geometric nature within the moduli space. Namely the bosonic sector in each side may be visualized as a geometric shape. The cross sectional shape of the DBI description has a gravity counterpart of the supertube shape. The moduli space of a supertube consists of the shape fluctuations and again each has its own counterpart in the supergravity side.

In fact, there is a regime where both descriptions may have their validity. Note that the radius squared of the circular supertubes is given by

$$R^2 = 2\pi g_s \ell_s^2 N_0 N_1 \frac{\ell_s}{L_z}, \quad (1)$$

where $N_0 = L_z Q_0$ and $N_1 = 2\pi Q_1$ are the numbers of D0 and F1 and L_z is the size of the compactified circle along which the axis direction of the supertube is wrapped. This is a new length scale introduced by supertubes and this estimation of the supertube size is valid unless $JL_z/(Q_0 Q_1) \ll 1$. The cross sectional area is quantized, which is related to the quantization of the angular momentum. Considering the case where $\gamma = L_z/(2\pi \ell_s)$ is of order one, the validity of supergravity description requires that

$$g_s N_0 N_1 \gg 1, \quad (2)$$

by $R \gg \ell_s$.

Since the energy of the supertubes are given by

$$M = \left(\frac{1}{2\pi \ell_s^2} N_1 + \frac{N_0}{g_s \ell_s L_z} \right) L_z = \frac{1}{g_s \ell_s} (g_s N_1 \gamma + N_0), \quad (3)$$

the Schwarzschild radius $R_S = (M g_s^2 \ell_s^8 / L_z)^{\frac{1}{6}}$ is

$$R_S = \ell_s [(g_s N_1 + N_0/\gamma) g_s / 2\pi]^{\frac{1}{6}}. \quad (4)$$

Thus, for $g_s N_0 N_1 \gg 1$, $R \gg R_S$, which may explain the regularity of the supertube solutions.

On the other hand, the DBI description has its validity in the decoupling limit of $\ell_s \rightarrow 0$ and $g_s \rightarrow 0$. Thus the overlapping region of the validity is given by the open-string decoupling limit,

$$\ell_s \rightarrow 0, \quad g_s \rightarrow 0, \quad (5)$$

while keeping the combination $g_s N_0 N_1$ large. Thus we have here the gravity and the supertube field theory correspondence in the overlapping regime of the validity. The decoupled field theory is the world-volume field theory of supertubes. In this limit, the field theory obtained by expanding DBI theory around the circular supertube background is eventually described by a peculiar 2+1 dimensional (noncommutative) Yang-Mills theory in the decoupling limit where $\ell_s \rightarrow 0$, which is equivalent to the matrix theory in a circular supertube background [2, 6, 14].

In this note, we would like to count the entropy of the geometries using the gravity/field theory correspondence and the structure of the phase moduli obtained in Ref. [27]. We shall be using basically the DBI action to count the degeneracy the states. This problem is in some sense already treated in Ref. [24], but the perspective and the emphasis on the geometric nature are the main differences.

We would like to first make it clear that we are basically counting the geometric fluctuations in the sense that, even in the field theory, we are counting the freedom of the shape fluctuation including other accompanying bosonic and fermionic degrees. This simpler version of the account of entropy using the shape fluctuation is presented first. The full derivation of the entropy in the decoupling limit is done via two different methods. One is the description of DBI action. Here we identify the infinite dimensional fermionic moduli space and count the entropy including all the fermionic fluctuations. In this case we use the near circular condition $q = (Q_0 Q_1 - J)/J \ll 1$ to simplify the calculation. The others are via the M-theory description of the M2-brane. In this M2 brane picture, we find that the near circular condition is not necessary for the counting.

As stated earlier, the quantization is necessary to get the correct expression and in this sense the decoupling limit is essential. At the end of the day, the decoupled theory in the same limit should lead to a unique theory in any paths.

The paper is organized as follows. In Sections 2 and 3, we review the phase moduli space of supertubes and count the cross sectional shape fluctuations. In Section 4, we explain the fermionic part of the moduli space using DBI description and count the full degeneracy in the near circular limit. In Sections 5, we count again the entropy using the M-theory. Section 6 is devoted to conclusions.

2 BPS Equations and Conserved Charges for a Closed Supertube

A tubular D2-brane with electric and magnetic fluxes on the world-volume becomes a closed supertube if it satisfies suitable BPS conditions. We first review these BPS equations and conserved charges for a closed supertube. This also serves to establish our notations.

The tubular D2-brane is embedded in the 10-dimensional flat space-time, and the world-volume is parametrized by (t, ϕ, z) . The pullback metric and the field strength on the D2-brane is written as

$$ds_{\text{pb}}^2 = -(1 - |\dot{\vec{x}}|^2)dt^2 + |\vec{x}'|^2 d\phi^2 + 2\dot{\vec{x}} \cdot \vec{x}' dt d\phi + dz^2, \quad (6)$$

$$F = E dt \wedge dz + B dz \wedge d\phi.$$

Here $\vec{x} = (x^1, \dots, x^8)$, $\dot{x} \equiv \frac{\partial x}{\partial t}$ and $x' \equiv \frac{\partial x}{\partial \phi}$. The cross section of the D2-brane is expressed as an arbitrary loop in \mathbb{R}^8 . The angle ϕ ($-\pi \leq \phi \leq \pi$) represents the direction along the loop

and z lies in the transverse direction. Thus we are considering only the configurations with the translational invariance in the z direction.

The bosonic part of the D2-brane DBI action is evaluated as

$$S = -T_2 \int dt d\phi dz \sqrt{(1 - |\dot{\vec{x}}|^2)(|\vec{x}'|^2 + \lambda^2 B^2) - \lambda^2 E^2 |\vec{x}'|^2 + (\dot{\vec{x}} \cdot \vec{x}')^2 - 2\lambda^2 E B \dot{\vec{x}} \cdot \vec{x}'}, \quad (7)$$

where a D2-brane tension T_2 and λ are written as $T_2 = \frac{1}{(2\pi)^2 \ell_s^3 g_s}$ and $\lambda = 2\pi \ell_s^2$, respectively, in terms of the string length ℓ_s and coupling g_s . Canonical momenta p_i and Π conjugate to x^i and A_z are written as

$$\begin{aligned} p_i &= -\frac{T_2}{\mathcal{L}} \left\{ \dot{x}_i (|\vec{x}'|^2 + \lambda^2 B^2) - (\dot{\vec{x}} \cdot \vec{x}') x_i' + \lambda^2 E B x_i' \right\}, \\ \Pi &= -\frac{T_2}{\mathcal{L}} \left\{ \lambda^2 E |\vec{x}'|^2 + \lambda^2 B \dot{\vec{x}} \cdot \vec{x}' \right\}, \end{aligned} \quad (8)$$

where \mathcal{L} is the Lagrangian density. The Hamiltonian density is given by

$$\begin{aligned} \mathcal{H} &= \sqrt{T_2^2 |\vec{x}'|^2 + T_2^2 \lambda^2 B^2 + |\vec{p}|^2 + \frac{\Pi^2}{\lambda^2}} \\ &= \sqrt{\left(\frac{\Pi}{\lambda} + T_2 \lambda B \right)^2 + \left(T_2 |\vec{x}'| - \frac{\Pi B}{|\vec{x}'|} \right)^2 + \left(|\vec{p}|^2 - \frac{\Pi^2 B^2}{|\vec{x}'|^2} \right)} \\ &\geq T \Pi + T_0 \frac{B}{2\pi}, \end{aligned} \quad (9)$$

where $T = \frac{1}{2\pi \ell_s^2}$ is the tension of the fundamental string and $T_0 = \frac{1}{\ell_s g_s}$ is the mass of a D0-brane. It can be verified that the third term in the square root in the second line is non-negative and vanishes when $\dot{\vec{x}} \propto \vec{x}'$, which is equivalent to $\dot{\vec{x}} = 0$ by suitable reparametrization of ϕ . Thus the Hamiltonian density is bounded from below by the mass density of fundamental strings and D0-branes, which precisely matches with the energy of the supertube. The equality in (9) is saturated when

$$\Pi B = T_2 |\vec{x}'|^2 \equiv T_2 \left(\frac{ds}{d\phi} \right)^2, \quad \dot{\vec{x}} = 0, \quad (10)$$

where $ds^2 = d\vec{x} \cdot d\vec{x}$ is the line element of \mathbb{R}^8 . These are the BPS conditions which must be satisfied by all supertubes.

The closed supertube carries two charges, which corresponds to those of fundamental strings and D0-branes, and angular momenta. Defining

$$Q_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \Pi, \quad Q_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi B, \quad (11)$$

we find that $2\pi Q_1 \in \mathbb{Z}$ is the number of fundamental strings dissolved in the D2-brane, and $Q_0 L_z \in \mathbb{Z}$ is that of D0-branes. The angular momenta are given as

$$L^{ij} = \frac{1}{2\pi} \oint_{-\pi}^{\pi} d\phi (x^i p^j - x^j p^i) = \frac{T_2}{\pi} \int dx^i \wedge dx^j, \quad (12)$$

where $i, j = 1, \dots, 8$. With the aid of the BPS equations (10), the canonical momenta are expressed as $p^i = T_2 x^{i'}$. The last equality in (12) is obtained by using this relation. Thus, when we fix the angular momenta, the area made by projecting the loop of the supertube onto (i, j) -plane should be preserved during the deformation in the flat directions.

The flat directions of supertubes of general shape with fixed fundamental string charge Q_1 , D0-brane charge Q_0 and angular momentum J are of our interest. They make the moduli space of the supertubes and are related to the number of gravity microstates. It was shown [27] that the perimeter $2\pi L$ of the supertubes is restricted by

$$\sqrt{J/T_2} \leq L \leq \sqrt{Q_0 Q_1 / T_2}. \quad (13)$$

The number of microstates allowed by this bound is the problem we are going to discuss in this paper.

3 A First Look on the Microstates of Supertubes

In order to get the idea how to count the microstates of the supertubes, let us first discuss the bosonic fluctuations $\epsilon(\phi)$, $a(\phi)$ and $b(\phi)$ around the circular background,

$$\zeta = x_1 + ix_2 = R(1 + \epsilon)e^{i\phi}, \quad \Pi = Q_1(1 + a), \quad B = Q_0(1 + b) \quad (14)$$

where ϵ , a and b are real. We consider the landscape of vacua having $Q_0 Q_1 > T_2 R^2$ with the area $\mathcal{A} = \pi R^2$ fixed. Introducing q defined by

$$Q_0 Q_1 = T_2 R^2 (1 + q), \quad (15)$$

we focus on the case of $q \ll 1$. The fluctuation can be expanded as¹

$$\epsilon(\phi) = \sum_{n \in \mathbb{Z}} \epsilon_n e^{in\phi}, \quad a(\phi) = \sum_{n \in \mathbb{Z}} a_n e^{in\phi}, \quad b(\phi) = \sum_{n \in \mathbb{Z}} b_n e^{in\phi}. \quad (16)$$

Here we should impose reality conditions $\epsilon_{-n} = \epsilon_n^\dagger$, $a_{-n} = a_n^\dagger$ and $b_{-n} = b_n^\dagger$.

The area given by

$$\mathcal{A} = \frac{1}{4} \int_0^{2\pi} \text{Im}(\zeta d\zeta^\dagger - \zeta^\dagger d\zeta) \quad (17)$$

is evaluated as

$$\mathcal{A} = \pi R^2 \left(1 + 2\epsilon_0 + \sum_{n \in \mathbb{Z}} |\epsilon_n|^2 \right). \quad (18)$$

¹As we shall see later on, a and b are mixed with the ϵ fluctuation. But for simplicity, we ignore this complication here. See also Ref. [27] for the detailed classical description of this mixing.

The conservation of the angular momentum implies that

$$2\epsilon_0 = - \sum_{n \in \mathbb{Z}} |\epsilon_n|^2. \quad (19)$$

The length $2\pi L$ of the curve

$$2\pi L = \int_0^{2\pi} |d\zeta| = R \int_0^{2\pi} d\phi \sqrt{(1+\epsilon)^2 + (\epsilon')^2} \quad (20)$$

may be expanded as

$$2\pi L = R \int_0^{2\pi} d\phi \left(1 + \frac{(2\epsilon + \epsilon^2 + (\epsilon')^2)}{2} - \frac{\epsilon^2}{2} + \dots \right) = 2\pi R \left(1 + \epsilon_0 + \sum_{n \in \mathbb{Z}} \frac{n^2}{2} |\epsilon_n|^2 + \dots \right). \quad (21)$$

The condition (19) may be used to eliminate ϵ_0 from (21) and we get

$$2\pi L = 2\pi R \left(1 + \sum_{n>1} (n^2 - 1) |\epsilon_n|^2 + \dots \right). \quad (22)$$

Here ϵ_n for $n > 0$ are our phase space variables and ϵ_0 is constrained by the condition (19). We are not interested in the translational mode ϵ_1 .

Since from (13)

$$R \leq L \leq \sqrt{Q_0 Q_1 / T_2}, \quad (23)$$

we get, using (15), a constraint

$$\sum_{n>1} (n^2 - 1) |\epsilon_n|^2 \leq q/2. \quad (24)$$

To compute the number of states in the volume (24), let us find out the canonical variables in the phase space. First we define the coordinate corresponding to the radius as $r(= R(1+\epsilon))$ which is real. From the action (7), we obtain the conjugate momentum

$$p_r(\phi) = T_2 r', \quad (25)$$

where use has been made of the fact that $E = 1/\lambda$ for the BPS states of our concern here and $\mathcal{L} = -T_2 \lambda B$. The relation (25) is a second class constraint and we should make Dirac quantization. After this procedure, one finds

$$[r(\phi), T_2 r'(\phi')] = \frac{i}{2} \delta(\phi - \phi') \delta(z - z'). \quad (26)$$

For the zero mode in the z direction, the above implies that

$$[\epsilon_m^\dagger, \epsilon_n] = \frac{1}{4\pi T_2 R^2 L_z n} \delta_{mn}, \quad (27)$$

where L_z is the length of the supertube in the z direction. Thus c_n defined by

$$\frac{\alpha}{\sqrt{n}} c_n^\dagger = \epsilon_n, \quad (28)$$

with $\alpha^2 = 1/(4\pi T_2 R^2 L_z)$ satisfies the commutation relation

$$[c_m, c_n^\dagger] = \delta_{mn}. \quad (29)$$

In terms of these variables, the constraint (24) becomes

$$\sum_{n=2}^{\infty} \left(n - \frac{1}{n}\right) |c_n|^2 \leq \frac{q}{2\alpha^2}. \quad (30)$$

Quantum mechanically, this condition is interpreted as

$$\sum_{n=2}^{\infty} \left(n - \frac{1}{n}\right) N_n \leq \frac{q}{2\alpha^2} \equiv s, \quad (31)$$

where the number operator N_n is defined by $c_n^\dagger c_n$.

Our task is now to evaluate the number of states restricted by (31). For large n , the $\frac{1}{n}$ in the bracket may be ignored, and the problem reduces to the well-known case of counting string states. It is given by

$$\mathcal{V} = \frac{\sqrt{2}}{4\pi\sqrt{s}} e^{\pi\sqrt{\frac{2}{3}}s}. \quad (32)$$

To get this, let us consider the following quantity [33]

$$G(w) = \sum_{n=0}^{\infty} d_n w^{nN_n} = \frac{1}{\prod_{n=1}^{\infty} (1 - w^n)} \equiv [f(w)]^{-1}. \quad (33)$$

This is related to the Dedekind eta function

$$\eta(\tau) = e^{i\pi\tau/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n \tau}), \quad (34)$$

which has the modular transformation formula

$$\eta(-1/\tau) = (-i\tau)^{1/2} \eta(\tau). \quad (35)$$

Applied to $f(w)$, this gives the Hardy-Ramanujan formula

$$f(w) = \left(\frac{-2\pi}{\log w}\right)^{1/2} w^{-1/2} \tilde{w}^{1/12} f(\tilde{w}^2), \quad (36)$$

where

$$\tilde{w} = e^{2\pi^2/\log w}. \quad (37)$$

One can then deduce the asymptotic formula for $w \rightarrow 1$ (or $\tilde{w} \rightarrow 0$)

$$f(w) \sim \left(\frac{-2\pi}{\log w} \right)^{1/2} \exp \left(\frac{\pi^2}{6 \log w} \right). \quad (38)$$

The degeneracy is obtained by

$$d_n = \oint \frac{G(w)}{w^{n+1}} \frac{dw}{2\pi i}. \quad (39)$$

Using the asymptotic expansion of $f(w)$, this can be estimated for large n by a saddle point evaluation. $G(w)$ grows rapidly for $w \rightarrow 1$, while if n is very large, w^{n+1} is very small for $w < 1$. There is a sharp saddle point for w near 1. The integrand (for the integration variable $\log w$)

$$\exp \left(-\frac{\pi^2}{6 \log w} - n \log w \right), \quad (40)$$

is stationary for $\log w \sim -\pi/\sqrt{6n}$. Evaluating (39) around this saddle point, we get [34]

$$d_n = \frac{1}{4\sqrt{3}n} e^{\pi\sqrt{\frac{2}{3}n}}, \quad (41)$$

in the large n . Integration of d_n up to s gives \mathcal{V} in (32).

Thus the entropy becomes

$$S = \ln \mathcal{V} = \pi \sqrt{\frac{q}{3\alpha^2}} = \pi \sqrt{\frac{4\pi}{3}} L_z (Q_0 Q_1 - J). \quad (42)$$

We note that this is the entropy from a single fluctuating boson around the supertube. In what follows we are going to extend this to supertubes with other modes.

4 Supertube solutions in DBI action and entropy

In this section we find exact BPS supertube solutions including fermion backgrounds using the DBI action of D2. The BPS solutions preserve $\frac{1}{4}$ supersymmetry. One may identify ‘fermionic flat directions’ of the classical solutions. We then give the quantization rules for the flat modes and we count the contributions to the entropy from fermions as well as bosons.

4.1 The solutions

We start by summarizing the supersymmetric DBI theory for the D2 in the notations and conventions of [35]. This action has gauge invariances coming from worldvolume diffeomorphism and the local kappa symmetry. Since the full gauge invariant action is complicated, we start from the action with gauge fixed kappa symmetry:

$$S = - \int d^3\sigma \sqrt{-\det[g_{\mu\nu} + F_{\mu\nu} - 2\bar{\lambda}\gamma_\mu\partial_\nu\lambda + (\bar{\lambda}\Gamma^M\partial_\mu\lambda)(\bar{\lambda}\Gamma_M\partial_\nu\lambda)]}, \quad (43)$$

where $\mu, \nu = t, \phi, z$ are the worldvolume indices, M is the R^{9+1} vector index, $g_{\mu\nu}$ is the pullback of 10-dimensional flat metric onto the world-volume, $\gamma_\mu = \Gamma_M \frac{\partial X^M}{\partial \sigma^\mu}$ is the induced gamma matrix,

and λ is the Majorana-Weyl fermion in the target space, where the Weyl condition is imposed by the gauge choice for the local kappa symmetry. We use the convention $\bar{\lambda} = \lambda^\dagger(-i\Gamma^0)$. As in Section 2, one should also include an overall coefficient T_2 , the D2-brane tension, and also replace F by $2\pi\ell_s^2 F$.

For later use, we summarize our gamma matrix conventions. The $SO(9,1)$ 32×32 Γ^M is expressed in terms of $SO(1,1)$ 2×2 gamma matrices and $SO(8)$ 16×16 gamma matrices as follows:

$$\mathbf{\Gamma}^0 = (i\sigma^2) \otimes \mathbf{1}_{16}, \quad \mathbf{\Gamma}^z = \sigma^1 \otimes \mathbf{1}_{16}, \quad \text{others } \sigma^3 \otimes \Gamma^i \quad (i = 1, 2, \dots, 8). \quad (44)$$

Γ^i 's are the $SO(8)$ spinors in a suitable representation: We use the convention that the former/latter 8 indices act on left/right chiral components, respectively. The last eight gamma matrices in (44) will be written as $\vec{\mathbf{\Gamma}} = \sigma^3 \otimes \vec{\Gamma}$, where the vector lives in R^8 . The chirality operator is defined as

$$\mathbf{\Gamma}_{11} = \mathbf{\Gamma}_0 \mathbf{\Gamma}_1 \cdots \mathbf{\Gamma}_8 \mathbf{\Gamma}_z = \sigma^3 \otimes \Gamma^9, \quad (45)$$

with the choice $\Gamma^9 = \text{diag}(\mathbf{1}_8, -\mathbf{1}_8)$. The Weyl condition on λ is chosen to be $\mathbf{\Gamma}_{11}\lambda = +\lambda$. With our convention (44) and (45), this chiral λ is written as

$$\lambda(\sigma) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \psi(\sigma) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \chi(\sigma) \end{bmatrix}, \quad (46)$$

where ψ, χ are 8-component $SO(8)$ spinors with Γ^9 eigenvalues ± 1 , respectively.

Let us make the *partial gauge fixing* for the world-volume diffeomorphism

$$T = t, \quad Z = z, \quad 0 \leq \phi < 2\pi, \quad \vec{x}(t, \phi, z) \in R^8, \quad (47)$$

i.e., we leave unspecified one scalar field in R^8 tangent to ϕ direction. This is harmless as far as we do the classical analysis.² In this gauge, the action (43) is written as

$$S = - \int d^3\sigma \sqrt{-\det \mathcal{M}_{\mu\nu}}, \quad (48)$$

where

$$\mathcal{M}_{\mu\nu} = \eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu \vec{x} \cdot \partial_\nu \vec{x} - 2\bar{\lambda}(\tilde{\gamma}_\mu + \vec{\Gamma} \cdot \partial_\mu \vec{x})\partial_\nu \lambda + (\bar{\lambda}\mathbf{\Gamma}^M \partial_\mu \lambda)(\bar{\lambda}\mathbf{\Gamma}_M \partial_\nu \lambda). \quad (49)$$

This is the final supersymmetric DBI action that we need. Note that, again due to the partial gauge fixing, $-\eta_{tt} = \eta_{zz} = 1$ but $\eta_{\phi\phi} = 0$; furthermore, $\tilde{\gamma}_t = \mathbf{\Gamma}_0$ and $\tilde{\gamma}_z = \mathbf{\Gamma}_z$ but $\tilde{\gamma}_\phi = 0$.

The solution we are looking for is independent of t and z , so we take $\vec{x}(\phi)$, $F = E(\phi)dt \wedge dz + B(\phi)dz \wedge d\phi$ and $\lambda(\phi)$ and insert them into the equations of motion. In this process, we may set all t, z derivatives of \vec{x} in the Lagrangian to zero, since terms containing these derivatives would not survive the equations of motion for \vec{x} . We can then rewrite $\sqrt{-\det \mathcal{M}_{\mu\nu}}$ as

$$\sqrt{|\vec{x}'|^2(1 - E^2) + B^2 + (1 - E^2)\{(\bar{\lambda}\mathbf{\Gamma}^M \lambda')^2 - 2\bar{\lambda}(\vec{\Gamma} \cdot \vec{x}')\lambda'\} + 2B\{E\bar{\lambda}\mathbf{\Gamma}_0 \lambda' - \bar{\lambda}\mathbf{\Gamma}_2 \lambda'\}}, \quad (50)$$

²However, when we consider quantization of near-circular supertubes, we should fix this extra gauge. There we will set this scalar equal to ϕ .

where the prime denotes ϕ derivative. Variation of this quantity in λ, \vec{x} and A_μ yields equations of motion. As we know that the supertube solution is obtained for $E = 1$, let us set $E = 1$ after variation in these fields, which simplifies the resulting equations drastically.

The variation of the Lagrangian in $\delta\bar{\lambda}$ (with $\delta\lambda$ since it is Majorana) gives (after setting $E = 1$)

$$\delta\mathcal{L} = -\frac{B}{\sqrt{B^2 + 2B\lambda(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_z)\lambda'}} [\delta\bar{\lambda}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_z)\lambda' + \bar{\lambda}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_z)\delta\lambda'] , \quad (51)$$

and the equation of motion

$$(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_z)\lambda' = 0 . \quad (52)$$

One can easily check that the full equations of motion are solved by $E = 1$ and (52) for any functions $B(\phi)$ and $\vec{x}(\phi)$. This is a simple generalization of the original supertube solution.

With the representation (44), (52) becomes $[\sigma^3 \otimes \mathbf{1}_{16}]\lambda' = \lambda'$. This means that the second term in (46) should be ϕ -independent:

$$\lambda(\phi) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \psi(\phi) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \chi \end{bmatrix} , \quad (\chi : \text{any constant spinor}). \quad (53)$$

Note that the fermionic part contains arbitrary functions of ϕ , an 8-component $SO(8)$ spinor $\psi(\phi)$ with positive chirality. The bosonic part has one from $B(\phi)$ plus seven gauge-invariant components from $\vec{x}(\phi)$. Below we shall show that $\chi = 0$ for the $\frac{1}{4}$ supersymmetry. Hence the fermionic part of supertubes also involves eight arbitrary functions of moduli fluctuation, which is expected from the number of remaining supersymmetries.

4.2 Supersymmetry

We now check whether the above solution preserves $\frac{1}{4}$ supersymmetry. The 32 supersymmetry parameters ϵ of type-IIA string theory are split into ϵ_\pm satisfying $\mathbf{\Gamma}_{11}\epsilon_\pm = \pm\epsilon_\pm$, respectively. The supersymmetry transformations, combined with compensating kappa transformation and world-volume diffeomorphism to restore the gauge, are

$$\begin{aligned} \delta\bar{\lambda} &= \bar{\epsilon}_+ + \bar{\epsilon}_-\gamma^{(z)} + \xi^\mu\partial_\mu\bar{\lambda}, \\ \delta\vec{x} &= (\bar{\epsilon}_+ - \bar{\epsilon}_-\gamma^{(z)})\vec{\mathbf{\Gamma}}\lambda + \xi^\mu\partial_\mu\vec{x}, \\ \delta A_\mu &= (\bar{\epsilon}_-\gamma^{(z)} - \bar{\epsilon}_+)(\tilde{\gamma}_\mu + \vec{\mathbf{\Gamma}} \cdot \partial_\mu\vec{x})\lambda \\ &\quad + (\frac{1}{3}\bar{\epsilon}_+ - \bar{\epsilon}_-\gamma^{(z)})\mathbf{\Gamma}_M\lambda \bar{\lambda}\mathbf{\Gamma}^M\partial_\mu\lambda + \xi^\nu\partial_\nu A_\mu + \partial_\mu\xi^\nu A_\nu, \end{aligned} \quad (54)$$

where $\xi^\mu = (\bar{\epsilon}_-\gamma^{(z)} - \bar{\epsilon}_+)\gamma^\mu\lambda$ (with $\mu = t, z$ only in the superscript) and the 32×32 matrix $\gamma^{(2)}$ in our case is (using $E=1$ and (52) to simplify the expression)

$$\gamma^{(z)} = -(\mathbf{\Gamma}_0\mathbf{\Gamma}_z + \mathbf{\Gamma}_{11}) \frac{\vec{x}' \cdot \vec{\mathbf{\Gamma}} - \vec{\mathbf{\Gamma}} \cdot (\bar{\lambda}\vec{\mathbf{\Gamma}}\lambda')}{B} - \mathbf{\Gamma}_{11}\mathbf{\Gamma}_0 . \quad (55)$$

Note that we have fixed the world-volume diffeomorphism only partially, i.e., $T = t$ and $Z = z$, so in the above definition we have two gauge-keeping parameters ξ^t and ξ^z but nothing like ξ^ϕ .

Let us start from $\delta\bar{\lambda}$. The last term is absent after inserting our solution, so we have

$$\delta\bar{\lambda} = \bar{\epsilon}_+ + \bar{\epsilon}_- \left\{ -(1 - \sigma^3) \frac{\vec{x}' \cdot \vec{\Gamma} - \vec{\Gamma} \cdot (\bar{\lambda} \vec{\Gamma} \lambda')}{B} + (i\sigma^2) \right\}, \quad (56)$$

where we have used $\bar{\epsilon}_- \Gamma_{11} = +\bar{\epsilon}_-$, and for all Pauli matrices $[\cdot \otimes \mathbf{1}_{16}]$ is implicit. Considering the chiralities of ϵ_{\pm} , we can write them in the following form:

$$\epsilon_+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \alpha_+ \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \beta_+ \end{bmatrix}, \quad \epsilon_- = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} \alpha_- \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \beta_- \end{bmatrix}. \quad (57)$$

In order for our solution to be supersymmetric, the first term in the curly bracket should vanish. This is true if $\beta_- = 0$. The cancellation of the remaining second term $\propto (i\sigma^2)$ with the $\bar{\epsilon}_-$ term requires $\alpha_+ = \alpha_-$ and $\beta_+ = 0$. Therefore we have $\frac{1}{4}$ of the ϵ components preserved; β_+ , β_- and $\alpha_+ - \alpha_-$ are broken.

Let us now consider the remaining transformations for bosonic fields. Using the expression (55) and the conditions $\beta_{\pm}=0$, $\alpha_+ = \alpha_- \equiv \alpha$, the scalar variation $\delta\vec{x}$ reduces to

$$\delta\vec{x} = -2i \begin{bmatrix} \alpha^\dagger & 0 \end{bmatrix} \vec{\Gamma} \begin{bmatrix} 0 \\ \chi \end{bmatrix}. \quad (58)$$

One can see from the above variation that all the supersymmetry is broken if the constant mode χ is non-zero. Therefore, as promised, we should set $\chi = 0$ in order to have $\frac{1}{4}$ -BPS deformation.

In order to check the last transformation, we should specify the vector potential giving rise to the field strength $F = dt \wedge dz + B(\phi) dz \wedge d\phi$. The simplest choice may be $A^{(1)} = t dz + B(\phi) z d\phi$. For later use, let us also consider an alternative choice. To this end, we decompose the magnetic field as

$$B_0 = \frac{1}{2\pi} \oint d\phi B(\phi), \quad b(\phi) = B(\phi) - B_0 \equiv -a'(\phi), \quad (59)$$

where $a(\phi)$ is a periodic function, due to the fact $\oint d\phi b(\phi) = 0$. Then we can choose the 1-form potential as

$$A^{(2)} = \{t + a(\phi)\} dz + z B_0 d\phi. \quad (60)$$

This form would be more convenient later, when we consider quantization. The two choices are related by a gauge transformation.

After inserting our supertube solution with $\chi = 0$, and turning on α only, the supersymmetry transformation (54) for gauge field becomes

$$\begin{aligned} \delta A_0^{(1)} &= \partial_t \{2i\alpha^\dagger \psi(\phi)\}, \\ \delta A_z^{(1)} &= 0, \\ \delta A_\phi^{(1)} &= \partial_\phi \{2i\alpha^\dagger \psi(\phi)\} + 2i\alpha^\dagger \psi(\phi) B(\phi). \end{aligned} \quad (61)$$

If the last term in the third line in (61) is absent, one can make a compensating gauge transformation and have $\frac{1}{4}$ supersymmetry. The subtle term proportional to $\psi(\phi) B(\phi)$ can be decomposed

into the ‘0-mode’ piece plus the remainder as

$$\overline{\psi B} \equiv \frac{1}{2\pi} \oint d\phi \psi(\phi) B(\phi) , \quad \{\psi(\phi) B(\phi)\}_r \equiv \psi(\phi) B(\phi) - \overline{\psi B}. \quad (62)$$

The ‘remainder’ piece $\{\psi(\phi) B(\phi)\}_r$ can be rewritten as $\partial_\phi[\cdot]$, where we have a well-defined periodic function of ϕ inside the square bracket. This can safely be compensated by a gauge transformation. The 0-mode piece can be written as $d\{2i\phi\alpha^\dagger \overline{\psi B}\}$, which is not a gauge transformation in general. To ensure supersymmetry, we require $\overline{\psi B} = 0$, which in turn implies that ψ 0-modes are expressed in terms of ψ and B nonzero modes.

4.3 Conserved charges

The electric displacement $\Pi(\sigma) = \frac{\partial \mathcal{L}}{\partial E}$ is obtained from (50) and setting $E = 1$ after differentiation:

$$\Pi(\sigma) = \frac{|\vec{x}'|^2}{B} - \bar{\lambda} \Gamma_0 \lambda' + \frac{1}{B} \left\{ (\bar{\lambda} \Gamma^M \lambda')^2 - 2\bar{\lambda} (\vec{\Gamma} \cdot \vec{x}') \lambda' \right\}. \quad (63)$$

One may show that all the higher-order terms in the curly bracket vanish for the supertube solution. The electric displacement reduces to

$$\Pi(\sigma) = \frac{|\vec{x}'|^2}{B} + i\psi^\dagger \psi', \quad (64)$$

for the supertubes.

The linear momentum density conjugate to \vec{x} becomes

$$\vec{p} = \vec{x}'. \quad (65)$$

Note that there is no correction from fermions. Consequently the field angular momentum also takes the same form as in the bosonic case,

$$L^{ij} = \frac{1}{2\pi} \oint d\phi (x^i p^j - x^j p^i) = \frac{1}{\pi} \int dx^i \wedge dx^j, \quad (66)$$

proportional to the cross section area of the tube. To obtain the conserved total angular momentum, we should add to it the spin angular momentum. The total angular momentum is then

$$J^{ij} = \frac{1}{\pi} \int dx^i \wedge dx^j - \frac{i}{4\pi} \oint d\phi B \psi^\dagger \Gamma^{ij} \psi, \quad (67)$$

where $\Gamma^{ij} \equiv \Gamma^{[i}\Gamma^{j]}$ is the anti-Hermitian $SO(8)$ Lorentz generators acting on spinors, which we understand as being reduced from 16×16 to 8×8 and acts on positive chirality subspace.

4.4 Quantization and entropy of near-circular supertubes

In order to quantize the modes identified before, we now fix the remaining diffeomorphism. As in the previous section, let us consider a small deformation from the circular tube with radius R_0 in the 1-2 plane. Then $x^1 + ix^2 = R(\phi)e^{i\phi}$ with $|R(\phi) - R_0| \ll R_0$, and $|x^i(\phi)| \ll R_0$ for

all $i = 3, 4, \dots, 8$. To identify the quantization conditions, we have to know the quadratic piece of the gauge-fixed Lagrangian. The full gauge fixed Lagrangian density becomes

$$\begin{aligned}
\mathcal{L} &= -\sqrt{\mathcal{L}_b^2 + \mathcal{L}_f^2} \\
\mathcal{L}_b^2 &= B^2 - (|\dot{\vec{x}}|^2 + \dot{R}^2)(|\vec{x}'|^2 + R^2 + (R')^2 + B^2) \\
&\quad + (\dot{\vec{x}} \cdot \vec{x}' + \dot{R}R')^2 - 2EB(\dot{\vec{x}} \cdot \vec{x}' + \dot{R}R') + (1 - E^2)(|\vec{x}'|^2 + R^2 + (R')^2) \\
\mathcal{L}_f^2 &= -2i \left\{ B^2 - B(\dot{\vec{x}} \cdot \vec{x}' + \dot{R}R') + (1 - E)(|\vec{x}'|^2 + R^2 + (R')^2) \right\} \psi^\dagger \dot{\psi} \\
&\quad - 2i \left\{ B(|\dot{\vec{x}}|^2 + \dot{R}^2) + (E - 1)(B + \dot{\vec{x}} \cdot \vec{x}' + \dot{R}R') \right\} \psi^\dagger \psi',
\end{aligned} \tag{68}$$

where vectors are in the six dimensional x^i space. With the choice of the vector potential $A_z = t + a$, the field strengths are given by $E = 1 + \dot{a}$, $B = B_0 - a'$. We expand this action up to quadratic order in $a(\phi, t)$, $r(\phi, t)$, $x^i(\phi, t)$, $\psi(\phi, t)$. The resulting quadratic Lagrangian density is

$$\mathcal{L}_2 = \frac{R_0}{B_0} \dot{a} \left\{ \frac{R_0}{B_0} a' + 2r \right\} + \dot{\vec{x}} \cdot \vec{x}' + \dot{r}r' + iB_0 \psi^\dagger \dot{\psi} + \frac{B_0^2 + R_0^2}{2B_0} \left\{ \dot{r}^2 + |\dot{\vec{x}}|^2 \right\} + \frac{R_0^2}{2B_0} \dot{a}^2. \tag{69}$$

Working within the 1/4 BPS phase moduli space only, the terms of quadratic time derivative may be dropped since the BPS states are time-independent.

The mode expansion for eight bosonic/fermionic fields are given as

$$\begin{aligned}
A_z = t + a(\phi, t) &= t + \sum_{n \neq 0} a_n(t) e^{-in\phi}, \quad R(\phi, t) = R_0 + r(\phi, t) = R_0 + \sum_{n \neq 0} r_n(t) e^{-in\phi}, \\
x^i(\phi, t) &= x_0^i + \sum_{n \neq 0} x_n^i(t) e^{-in\phi}, \quad \psi(\phi, t) = \sum_{n \neq 0} \psi_n(t) e^{-in\phi},
\end{aligned} \tag{70}$$

with $a_{-n} = a_n^\dagger$, $r_{-n} = r_n^\dagger$ and $x_{-n}^i = (x_n^i)^\dagger$. The transverse center of mass positions x_0^i would not affect the following analysis, so we will neglect them. $\psi(\phi, t)$ and ψ_n 's carry eight components. Inserting the mode expansions into (69) and integrating over ϕ and z (for the zero mode in the z direction), we get the Lagrangian

$$L_2 = 2\pi L_z \sum_{n \neq 0} \left[in r_n^\dagger \dot{r}_n + \frac{R_0}{B_0} \left(in \frac{R_0}{B_0} a_n^\dagger + 2r_n^\dagger \right) \dot{a}_n + in \vec{x}_n^\dagger \cdot \dot{\vec{x}}_n + iB_0 \psi_n^\dagger \dot{\psi}_n \right]. \tag{71}$$

Introducing

$$X_{n\pm} \equiv r_n \mp i \frac{R_0}{B_0} a_n, \tag{72}$$

the Lagrangian becomes

$$L_2 = 2\pi L_z \sum_{n=1}^{\infty} \left[i(n+1) X_{n+}^\dagger \dot{X}_{n+} + i(n-1) X_{n-}^\dagger \dot{X}_{n-} + 2in \vec{x}_n^\dagger \cdot \dot{\vec{x}}_n + 2iB_0 \psi_n^\dagger \dot{\psi}_n \right]. \tag{73}$$

After the Dirac quantization procedure, the resulting commutation relations read

$$\begin{aligned} [X_{m\pm}, X_{n\pm}^\dagger] &= \frac{1}{2\pi L_z(n\pm 1)} \delta_{m,n}, \\ [x_m^i, x_n^{j\dagger}] &= \frac{1}{2\pi L_z(2n)} \delta_{m,n} \delta^{i,j}, \\ \{\psi_m, \psi_n^\dagger\} &= \frac{1}{2\pi L_z(2B_0)} \delta_{m,n}, \end{aligned} \quad (74)$$

with all the other commutators vanishing ($i, j = 3, 4, \dots, 8$, $m, n = 1, 2, \dots$). Note that the radius r_n and gauge field a_n modes mix nontrivially in the commutation relation. Special remark for X_{1-} is in order: The above relation is meaningless for X_{1-} . This is natural since the dipole deformation of radius $R(\phi)$ is nothing but the translation of supertube along 1-2 plane [27]. So we expect that there are true zero modes having the quadratic time derivative terms only for their kinetic part.³ We are not interested in this translational zero mode.

The conserved charges are expressed as

$$J_{12} = R_0^2 + \sum_{n>0} \left\{ 2|r_n|^2 - iB_0\psi_n^\dagger \Gamma_{12} \psi_n \right\}, \quad (75)$$

$$Q_0 Q_1 = R_0^2 + 2 \sum_{n>0} \left\{ \left| r_n - i n a_n \frac{R_0}{B_0} \right|^2 + n^2 |r_n|^2 + |\vec{x}_n|^2 + nB_0 |\psi_n|^2 \right\}, \quad (76)$$

where $|A|^2 = A^\dagger A$ is our ordering convention. The two charges commute, as they should. The first expression (75) determines R_0^2 in terms of oscillators and J . Inserting this into (76), we obtain

$$Q_0 Q_1 - J = \sum_{n>0} \left\{ n(n+1) |X_{n+}|^2 + n(n-1) |X_{n-}|^2 + 2n^2 |\vec{x}_n|^2 + 2nB_0 |\psi_n|^2 + iB_0 \psi_n^\dagger \Gamma_{12} \psi_n \right\}. \quad (77)$$

Here we choose a basis for the spinor ψ such that $i\Gamma_{12}$ is diagonal with four ± 1 eigenvalues with corresponding modes $\psi_{n\pm}^\alpha$, ($\alpha = 1, 2, 3, 4$). Furthermore, we normalize the oscillators in the canonical way as follows:

$$X_{n\pm} = \frac{1}{\sqrt{2\pi L_z(n\pm 1)}} Y_{n\pm}, \quad \vec{x}_n = \frac{1}{\sqrt{2\pi L_z(2n)}} \vec{y}_n, \quad \psi_{n\pm}^\alpha = \frac{1}{\sqrt{2\pi L_z(2B_0)}} \Psi_{n\pm}^\alpha \quad (78)$$

with $n > 0$. The new oscillators satisfy the commutation relations $[Y_{n\pm}, Y_{n\pm}^\dagger] = [y_n^i, y_n^{i\dagger}] = \{\Psi_{n\pm}^\alpha, \Psi_{n\pm}^{\alpha\dagger}\} = 1$. Then we can rewrite (77) as

$$\begin{aligned} 2\pi L_z(Q_0 Q_1 - J) &= \sum_{n>0} \left\{ n |Y_{n+}|^2 + n |Y_{n-}|^2 + n |\vec{y}_n|^2 + (n + \frac{1}{2}) |\Psi_{n+}^\alpha|^2 + (n - \frac{1}{2}) |\Psi_{n-}^\alpha|^2 \right\} \\ &= \sum_{n>0} \left\{ \sum_{I=1}^8 n N_n^I + \sum_{\alpha=1}^4 \left[(n + \frac{1}{2}) N_{n+}^\alpha + (n - \frac{1}{2}) N_{n-}^\alpha \right] \right\}, \end{aligned} \quad (79)$$

³In the gravity description, this mode is like the freedom of translating black holes or supertubes.

where the last expression contains 8 classes of bosonic number operators N_n^I ($I = 1, 2, \dots, 8$) and 4 classes of fermionic number operators $N_{n\pm}^\alpha$ with \pm spins. $2\pi L_z(Q_0 Q_1 - J)$ may take half-integer eigenvalues.

The entropy can be counted by considering the generating function $\text{tr}(\omega^{2N}) = \sum_{n=0}^{\infty} d_n \omega^n$ where N is the number operator (79), and obtaining the degeneracy d_n with $\frac{n}{2} = 2\pi L_z(Q_0 Q_1 - J)$ being a large half-integer. One has

$$\text{tr}(\omega^{2N}) = \left(\prod_{m=1}^{\infty} (1 - \omega^{2m}) \right)^{-8} \left(\prod_{m=1}^{\infty} (1 + \omega^{2m+1}) \right)^4 \left(\prod_{m=1}^{\infty} (1 + \omega^{2m-1}) \right)^4. \quad (80)$$

The saddle-point evaluation of the degeneracy

$$d_n = \frac{1}{2\pi i} \oint d\omega \frac{\text{tr}(\omega^{2N})}{\omega^{n+1}}, \quad (81)$$

requires the behavior of the functions $f_{\pm}(z) \equiv \prod_{n=1}^{\infty} (1 \pm z^n)$ near $z \approx 1^-$. Up to the prefactors that we do not need, we have

$$f_+(z) \sim \exp \left[\frac{\pi^2}{12} \frac{1}{1-z} \right], \quad f_-(z) \sim \exp \left[-\frac{\pi^2}{6} \frac{1}{1-z} \right]. \quad (82)$$

We also note that, as long as we are interested in saddle point evaluation for large $2\pi L_z(Q_0 Q_1 - J)$, we may regard $\omega^{2n\pm 1}$ in (80) as ω^{2n} . Using the above formulae and noting that $\log \omega \approx -(1-\omega)$ for $\omega \approx 1^-$, one can see that (81) gets dominant contributions near $\log \omega \approx -\sqrt{\frac{\pi^2(8+4)}{12(n+1)}} \approx -\sqrt{\frac{\pi^2}{n}}$, where $c_B = 8$ and $c_F = 4$ denotes boson/fermion contributions, respectively. The result is

$$d_n \sim \exp \left[2\pi \sqrt{(c_B + c_F) \frac{n}{12}} \right] = \exp [2\pi \sqrt{n}], \quad (c_B = 2c_F = 8), \quad (83)$$

up to the prefactor, which is a suitable power of n . Inserting $n = 4\pi L_z(Q_0 Q_1 - J)$, we get the final expression for the supertube entropy

$$S = \log(d_n) = 4\pi \sqrt{\pi L_z(Q_0 Q_1 - J)}. \quad (84)$$

As expected, this is $\sqrt{c_B + c_F} = \sqrt{12}$ times the entropy (42) from one boson.

One may consider more general case of the multiple circular supertubes carrying $SO(8)$ Cartans $J_a = J_{2a-1, 2a}$ for $a = 1, 2, 3, 4$. The relevant background is described by

$$x_{2a-1} + ix_{2a} = R_a e^{i\phi} \quad (85)$$

with

$$\Pi_0 B_0 = T_2 \sum_{a=1}^4 R_a^2. \quad (86)$$

The corresponding angular momentum becomes $J_a = T_2 R_a^2$. Repeating the above analysis, one may get straightforwardly

$$S = 4\pi \sqrt{\pi L_z \left(Q_0 Q_1 - \sum_{a=1}^4 |J_a| \right)}, \quad (87)$$

in an appropriate near circular limit.

5 Microstates in Supermembrane Picture

In this section we derive the entropy of supertube with charges Q_1 , Q_0 and angular momentum J_a ($a = 1, \dots, 4$) from the 11-dimensional M-theory point of view including the contribution from bosons and fermions. We study equations of motion for a supermembrane with winding number and momentum along the 11-th direction, which preserves 1/4 supersymmetry. This approach gives a simple derivation of BPS equations because fields on the supermembrane are only 11 bosons $X^M = (t, z, x^i (i = 1, \dots, 8), x^{\natural})$ and a Majorana fermion Θ , which denote the embedding of the supermembrane into the superspace.

Let us investigate BPS equations for the supermembrane. These are obtained by analyzing the Killing spinor equations of [36]

$$\begin{aligned}\delta X^M &= i\bar{\epsilon}\Gamma^M\Theta + i\bar{\Theta}\Gamma^M(1 + \Gamma)\kappa = 0, \\ \delta\Theta &= \epsilon + (1 + \Gamma)\kappa = 0,\end{aligned}\tag{88}$$

where Γ^M are 11-dimensional gamma matrices, and Γ is defined as

$$\Gamma = \frac{1}{3!\sqrt{-\det P[G(X^M, \Theta)]_{ab}}} \epsilon^{abc} \partial_a \Pi^L \partial_b \Pi^M \partial_c \Pi^N \Gamma_{LMN}.\tag{89}$$

Here $P[G(X^M, \Theta)]_{ab}$ is the induced metric on the world-volume, and Π^M are super invariant 1-forms, $\Pi^M = dX^M - i\bar{\Theta}\Gamma^M d\Theta$. Note that Γ satisfies $\Gamma^2 = 1$ and $\text{tr } \Gamma = 0$. By using the former property, κ can be eliminated and the Killing spinor equations simply become

$$\bar{\Theta}\Gamma^M\epsilon = 0,\tag{90}$$

$$(1 - \Gamma)\epsilon = 0.\tag{91}$$

Since we are considering the supermembrane which corresponds to the supertube, the solution should be 1/4 supersymmetric [12]:

$$\epsilon = \frac{1 + \Gamma_{t\natural z}}{2} \frac{1 + \Gamma_{t\natural}}{2} \epsilon_0,\tag{92}$$

where ϵ_0 is an arbitrary Majorana spinor.

We then find that Eq. (90) has the solution (92) iff

$$\Theta = \frac{1 - \Gamma_{t\natural z}}{2} \frac{1 + \Gamma_{t\natural}}{2} \Theta_0,\tag{93}$$

where Θ_0 is an arbitrary Majorana spinor. We see that Θ has 8 real components. It is easy to verify the following relations:

$$\begin{aligned}\bar{\Theta}\Gamma^z\Theta &= \bar{\Theta}\Gamma^i\Theta = \bar{\Theta}\Gamma_{t\natural}\Theta = \bar{\Theta}\Gamma_{i\natural}\Theta = \bar{\epsilon}\Gamma_{t\natural}\Theta = \bar{\epsilon}\Gamma_{z\natural}\Theta = 0, \\ \Gamma^t\Theta &= \Gamma^{\natural}\Theta = \Gamma^{z\natural}\Theta = \Gamma_{tz}\Theta.\end{aligned}\tag{94}$$

Next we consider Eq. (91). The world-volume coordinates on the supermembrane are identified with (τ, ϕ, z) , and we assume that x^i , x^{\natural} and Θ depend only on τ and ϕ . Then the superinvariant 1-forms Π^M are expressed as

$$\begin{aligned}\Pi^M &= \dot{\Pi}^M d\tau + \Pi^{M'} d\phi, \quad (M = t, i, \natural), \\ \Pi^z &= dz + \dot{\Pi}^z d\tau + \Pi^{z'} d\phi,\end{aligned}\tag{95}$$

where $\dot{\Pi}^M = \dot{X}^M - i\bar{\Theta}\Gamma^M\dot{\Theta}$ and $\Pi^{M'} = X^{M'} - i\bar{\Theta}\Gamma^M\Theta'$. Notations \dot{X} and X' represent $\frac{\partial X}{\partial \tau}$ and $\frac{\partial X}{\partial \phi}$, respectively. Then Γ is written as

$$\begin{aligned}\Gamma &= \frac{1}{\sqrt{X}} \left\{ (\dot{\Pi}^t \Pi^{i'} - \Pi^{t'} \dot{\Pi}^i) \Gamma_{tiz} + (\dot{\Pi}^{\natural} \Pi^{i'} - \Pi^{\natural'} \dot{\Pi}^i) \Gamma_{\natural iz} + (\dot{\Pi}^t \Pi^{\natural'} - \dot{\Pi}^{\natural} \Pi^{t'}) \Gamma_{t\natural z} + \dot{\Pi}^i \Pi^{j'} \Gamma_{ijz} \right\}, \\ \sqrt{X} &= \sqrt{-(-\dot{\Pi}^t{}^2 + \dot{\Pi}^i{}^2 + \dot{\Pi}^{\natural}{}^2)(-\Pi^{t'2} + \Pi^{i'2} + \Pi^{\natural'2}) + (-\dot{\Pi}^t \Pi^{t'} + \dot{\Pi}^i \Pi^{i'} + \dot{\Pi}^{\natural} \Pi^{\natural'})^2},\end{aligned}\tag{96}$$

where we have defined the volume factor \sqrt{X} . From these equations, we find that Eq. (92) becomes the solution of (91) when

$$\dot{\Pi}^i = k \Pi^{i'}, \quad \dot{\Pi}^t - k \Pi^{t'} = \dot{\Pi}^{\natural} - k \Pi^{\natural'},\tag{97}$$

where k is a constant. Later we set $k = -1$. Therefore the BPS equations of the supermembrane corresponding to the supertube are given by (93) and (97).

Now that we have obtained the BPS equations which minimize the energy of the supermembrane, our next task is to derive conditions to fix two charges Q_1 , Q_0 and angular momenta. The two charges are winding number and momentum along the x^{\natural} direction, so the conditions are written as

$$\begin{aligned}Q_1 &= \frac{1}{2\pi} \oint_{-\pi}^{\pi} d\phi \frac{x^{\natural'}}{2\pi R_{11}}, \\ Q_0 &= \oint_{-\pi}^{\pi} d\phi R_{11} p^{\natural}, \\ J_{ij} &= \frac{1}{2\pi} \oint_{-\pi}^{\pi} d\phi (x^i p^j - x^j p^i - \frac{1}{2} S \Gamma^{ij} \Theta),\end{aligned}\tag{98}$$

where R_{11} is the radius of the 11th circle, and p_{\natural} and S_{α} are the conjugate momenta to x^{\natural} and Θ^{α} , respectively. Note that $2\pi Q_1$ and $Q_0 L_z$ are integers. From now on we identify τ and ϕ with $t = 2\pi Q_1 R_{11} \tau$ and $x^{\natural} = 2\pi Q_1 R_{11} \phi$, respectively. With this choice, the first equation in (98) is trivially satisfied.

To explicitly write down the other conditions, we need the conjugate momenta. These are calculated from the supermembrane action:

$$\begin{aligned}S_{M2} &= S_{NG} + S_{WZ}, \\ S_{NG} &= -T_2 \int d\tau d\phi dz \sqrt{X}, \\ S_{WZ} &= T_2 \int d\tau d\phi dz (i x^{\natural'} \bar{\Theta} \Gamma^t \dot{\Theta} - i x^{\natural} \bar{\Theta} \Gamma^t \Theta' + i t \bar{\Theta} \Gamma^t \Theta').\end{aligned}\tag{99}$$

We have reduced the degrees of freedom of Θ by using (93), and assumed the same ansatz as (95). By imposing the BPS condition (97), the conjugate momenta are given by

$$p^i = -T_2 x^{i'}, \quad (100)$$

$$p^{\natural} = \frac{T_2 |\vec{x}'|^2}{x^{\natural'}} - 2iT_2 \bar{\Theta} \Gamma^t \Theta', \quad (101)$$

$$S = 2iT_2 x^{\natural'} \bar{\Theta} \Gamma^t. \quad (102)$$

The conditions for fixing charges and angular momenta are then written as

$$\begin{aligned} \frac{Q_1 Q_0}{T_2} &= \frac{1}{2\pi} \oint d\phi (|\vec{x}'|^2 - 2ix^{\natural'} \bar{\Theta} \Gamma^t \Theta'), \\ \frac{J_a}{T_2} &= \frac{1}{2\pi} \oint d\phi (-x^{2a-1} x^{2a'} + x^{2a} x^{2a-1'} - ix^{\natural'} \bar{\Theta} \Gamma^t \Gamma^{2a-1, 2a} \Theta), \end{aligned} \quad (103)$$

where $a = 1, 2, 3, 4$ labels the $SO(8)$ Cartans. These are the constraints obtained by fixing Q_1 , Q_0 and angular momenta.

Now let us consider the quantization of x^i and Θ . From the BPS equations $(\partial_\tau + \partial_\phi)x^i = 0$, x^i contain only right moving modes,

$$x^i(\tau, \phi) = x_0^i + \frac{p_0^i}{2\pi L_z T_2}(\tau - \phi) + \frac{i}{\sqrt{4\pi T_2 L_z}} \sum_{m \neq 0} \frac{\sqrt{|m|}}{m} x_m^i e^{-im(\tau - \phi)}. \quad (104)$$

Since x^i and p^i are related as (100), we need the Dirac quantization of the constrained system. After some calculations, we obtain the commutation relations $[x^i(\phi), p^j(\phi')] = \frac{i}{2L_z} \delta^{ij} \delta(\phi - \phi')$, and hence

$$[x_m^i, x_n^{j\dagger}] = \delta^{ij} \delta_{mn}. \quad (105)$$

The Majorana fermion Θ is also treated similarly. From the equations of motion obtained by (99) and BPS equations (97), we only need the right moving modes and $\Theta_\alpha (\alpha = 1, \dots, 8)$ are expanded as

$$\Theta_\alpha = \frac{1}{\sqrt{8\pi T_2 x^{\natural'} L_z}} \sum_m \Theta_{m\alpha} e^{-im(\tau - \phi)}. \quad (106)$$

Since Θ_α and S^α are related as (102), after the calculation of Dirac brackets we obtain $\{\Theta_\alpha, S^\beta\} = \frac{i}{2L_z} \delta_\alpha^\beta \delta(\phi - \phi')$, and hence

$$\{\Theta_{m\alpha}, \Theta_n^{\dagger\beta}\} = \delta_\alpha^\beta \delta_{mn}, \quad (107)$$

where $\Theta^\dagger = -i\Theta^T C^{-1} \Gamma^t$.

After the quantization, the constraints (103) reduce to

$$\begin{aligned} 2\pi L_z Q_0 Q_1 &= \sum_{i=1}^8 \sum_{m=1}^{\infty} m x_m^{i\dagger} x_m^i + \sum_{\alpha=1}^8 \sum_{m=1}^{\infty} m \Theta_m^{\dagger\alpha} \Theta_{m\alpha}, \\ 2\pi L_z J_a &= i \sum_{m=1}^{\infty} (x_m^{2a-1} x_m^{2a\dagger} - x_m^{2a} x_m^{2a-1\dagger}) - i \sum_{\alpha=1}^8 \sum_{m=1}^{\infty} \frac{1}{2} \Theta_m^{\dagger\alpha} \Gamma^{2a-1, 2a} \Theta_{m\alpha}, \end{aligned} \quad (108)$$

Note that $2\pi L_z Q_0 Q_1$ is an integer. The volume of the phase moduli space is obtained exactly by counting the configurations of x^i and Θ_α which satisfy the above constraints.

We choose the spinor eigen basis diagonalizing $i\Gamma^{2b-1,2b}$ ($b = 1, 2, 3$). One has

$$i\Gamma^{2b-1,2b}\Theta_{n,\vec{s}} = s_b\Theta_{n,\vec{s}}, \quad (109)$$

where $\vec{s} = (s_1, s_2, s_3)$ with $s_b = \pm 1$. Since $\prod_{a=1}^4 \Gamma^{2a-1,2a}\Theta_{n,\vec{s}} = \Theta_{n,\vec{s}}$, the eigenvalue s_4 of $i\Gamma^{78}$ is given by $s_1 s_2 s_3$.

The combination of the constraints in (108) give

$$\begin{aligned} N &= 2\pi L_z (Q_0 Q_1 - \sum_a J_a) \\ &= \sum_{m=1}^{\infty} \left\{ \sum_{a=1}^4 \left((m+1)A_{am}^\dagger A_{am} + (m-1)B_{am}^\dagger B_{am} \right) + \sum_{\vec{s}} \left(m + \frac{1}{2} \sum_{a=1}^4 s_a \right) \Theta_{m\vec{s}}^\dagger \Theta_{m\vec{s}} \right\}, \end{aligned} \quad (110)$$

where we have defined

$$\begin{aligned} A_{am} &= \frac{1}{\sqrt{2}}(x_m^{2a-1} + ix_m^{2a}), & B_{am} &= \frac{1}{\sqrt{2}}(x_m^{2a-1} - ix_m^{2a}), \\ [A_{am}, A_{bm}^\dagger] &= \delta_{ab}, & [B_{am}, B_{bm}^\dagger] &= \delta_{ab}. \end{aligned} \quad (111)$$

The second constraint in (108) is written as

$$\sum_{m=1}^{\infty} (B_{am}^\dagger B_{am} - A_{am}^\dagger A_{am}) - \frac{1}{2} \sum_{\vec{s}} \sum_{m=1}^{\infty} s_a \Theta_{m\vec{s}}^\dagger \Theta_{m\vec{s}} = 2\pi L_z J_a. \quad (112)$$

We can consider this determines B_{a1} mode which is absent from (110). Thus the number of microstates can be counted by taking only the constraint (110) into account.

We consider the case $N \gg 1$. As in the previous sections, let us compute the partition function

$$\begin{aligned} G(w) &= \text{tr } w^N = \sum_{n=0}^{\infty} d_n w^n \\ &= \frac{2(1+w^{-1})(1-w)^4}{(1+w)(1+w^2)} \prod_{m=1}^{\infty} \frac{(1+w^m)^8}{(1-w^m)^8} \\ &= \frac{(1-w)^4(1+w^{-1})}{2^3(1+w)(1+w^2)} \frac{\vartheta_{10}^4(0, \tau)}{\eta^{12}(\tau)} \\ &= \frac{(1-w)^4(1+w^{-1})}{2^3(1+w)(1+w^2)} \left(-\frac{\ln w}{2\pi} \right)^4 \frac{\vartheta_{01}^4(0, -1/\tau)}{\eta^{12}(-1/\tau)}, \end{aligned} \quad (113)$$

where we have used the eta function defined in Eq. (34) and

$$\begin{aligned} \vartheta_{10}(0, \tau) &= 2w^{\frac{1}{8}} \prod_{m=1}^{\infty} (1-w^m)(1+w^m)^2, \\ \vartheta_{01}(0, \tau) &= \prod_{m=1}^{\infty} (1-w^m)(1-w^{m-1/2})^2, \end{aligned} \quad (114)$$

with $w = e^{2\pi i\tau}$, and the modular transformations of the eta (35) and

$$\vartheta_{10}(0, -1/\tau) = \sqrt{-i\tau} \vartheta_{01}(0, \tau). \quad (115)$$

The result (113) gives the asymptotic formula

$$G(w) \sim \pi^4 \left(-\frac{\log w}{2\pi} \right)^8 \exp \left(-\frac{2\pi^2}{\log w} \right), \quad (116)$$

for $w \sim 1^-$. The saddle point approximation enables us to derive the final result for the degeneracy for $N = n$ as

$$d_n = \frac{1}{2\pi i} \oint \frac{G(w)}{w^{n+1}} dw \sim e^{2\pi\sqrt{2n}}. \quad (117)$$

In this way we obtain the entropy

$$S = \log d_N \sim 4\pi \sqrt{\pi L_z \left(Q_0 Q_1 - \sum_{a=1}^4 |J_a| \right)}, \quad (118)$$

in agreement with (87).

We would like to emphasize that we have not used the near circular condition $(Q_0 Q_1 - J)/J \ll 1$ anywhere for the evaluation of the entropy in this section. So the entropy formula (118) will be valid beyond the near circular limit. Let us confirm this for the case of $\Theta = 0$ and $J_a = 0$ ($a = 2, 3, 4$) for simplicity. The expectation value of the radius squared is given by

$$R^2 \equiv \frac{1}{2\pi} \oint d\phi |\vec{x}|^2 = \frac{1}{2\pi L_z T_2} \sum_{m=1}^{\infty} \sum_{a=1}^4 \left(\frac{1}{m} A_{am}^\dagger A_{am} + \frac{1}{m} B_{am}^\dagger B_{am} \right). \quad (119)$$

As discussed in the introduction, we need to have $R_S \ll R$ for the validity of our counting of microstates. The value (119) of R should be estimated under the constraints (108) or (110) and (112). The constraint (112) tells us that we must excite certain amount of B_{am} and A_{am} , and R is smaller if those with larger m are excited, but there is an upper limit on possible m from the first equation in (108). We thus find that the minimum of R is attained when we excite $2\pi L_z J_1$ of B_{1m} for $m \sim Q_0 Q_1 / J_1$, giving

$$R \sim \sqrt{\frac{J_1}{Q_0 Q_1}} \sqrt{\frac{J_1}{T_2}}. \quad (120)$$

Thus, as long as $J_1 / Q_0 Q_1$ is not very small, $R_S \ll R$ can be satisfied for large J_1 and therefore the entropy formula (118) is valid.

6 Conclusions

In this paper we have presented two approaches to the counting of the number of microstates for supertubes specified by the F1 and D0 charges and angular momenta, and derived consistent entropy formula.

There are corresponding supergravity microstates and, thus, we count the degeneracy of the geometries with the asymptotic geometry and charges fixed. The correspondence demonstrates the existence of the quantized microstates specified by the distinguishable supergravity fields. Thus although we do not know how to do precisely, there must be a clear way to sum over geometries with an appropriate measure. This has been suspected in many cases including the thermal AdS/CFT correspondence [37], where one has competing contributions from the AdS Schwarzschild black hole and the Euclidean AdS geometry of temporal circle size related to the inverse temperature.

Indeed the related black hole entropy may be understood from the microstates. Since the horizon area of the supergravity supertubes are zero in any cases, the situation here is rather confusing. However, the proposal of Sen [38] may be applied and the stretched horizon area of the rotationally symmetric solution may be shown to agree to the entropy [32].

The situation of D1-D5-P [26] which is related to F1-D0-D4 by a U-duality is different. [D1 (5)-D5 (56789)-P (5) where the numbers in the parenthesis represent momentum direction or extending directions, is related to F1 (5)-D0-D4 (6789) by the successive transformations of S, T5, S, T56789.] The rotationally symmetric black hole solution of D1-D5-P has a nonvanishing horizon area and the corresponding entropy can be explained by the CFT counting [40].

When we add J_{12} angular momentum to the F1 (5)-D0-D4 (6789)-D2 ($5\theta_{12}$), the configuration describes the supertubes intersecting with D4-branes, which preserve four real supersymmetries. By the same U-duality transformation, the above is related to the D1 (5)-D5 (56789)-P (5)-KK5 ($6789\theta_{12}$) [31] where θ_{12} represents that the KK monopole or the D2 form a curve in the (12) plane. Similarly J_{34} may be added too.

Since the supertube ending on D4 in Refs. [11, 15, 18, 39] has angular momenta in (6789) plane only, e.g. F1 (5)-D0-D4 (6789)-D2 ($5\theta_{67}$), the above configurations of the curve in (1234) plane are different in their expansion directions of D2 and have not been found in the field theory description.

Considering the supertubes suspended between two D4-branes of large separations, the corresponding entropy is expected as $S = \frac{4\pi}{\sqrt{2}} \sqrt{\pi L(Q_0 Q_1 - \sum |J|)}$, where the sum is over the $SO(4)$ Cartans in (6789) plane. The curve cannot escape to the (1234) plane because the supertube ends on D4-branes. Thus only four arbitrary bosonic fluctuations remain. Furthermore they preserve four real supersymmetries and the number of arbitrary fermionic fluctuations should be reduced to four. Hence one has the $1/\sqrt{2}$ factors. (The half factor for the numbers of degrees goes inside of the square root.)

For many D4-branes, the above formula would have a straightforward generalization. For the supertubes connecting D4-branes, one may have in principle five independent charges; D0, F1, D4 and two Cartans of the angular momenta.

For these cases, one has clean examples of the gravity microstates, black hole solutions whose horizon area reproduces the entropy, and the corresponding field theory description of

the microstates. But the detailed and complete construction awaits more endeavors.

Finally, the formula for the cross sectional area $\mathcal{A} \sim g_s \ell_s^2 N_0 N_1$ in (1) is reminiscent of the quantum foam in Ref. [41]. Since the counting and the partition function may be also related to the black hole partition function [42], there seems to be some connections of the microstates to the quantum foam in Ref. [41]. Any clue in this direction will be very interesting.

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